

The Axiomatic System of Geometry

Back to the Beginning

SUGGESTED LEARNING STRATEGIES: Close Reading, Quickwrite, Think/Pair/Share, Vocabulary Organizer, Interactive Word Wall

My Notes

Geometry is an **axiomatic system**. That means that from a small, basic set of agreed-upon assumptions and premises, an entire structure of logic is devised. Many interactive computer games are designed with this kind of structure. A game may begin with basic set of scenarios. From these scenarios, a gamer can devise tools and strategies to win the game.

In geometry, it is necessary to agree on clear-cut meanings, or definitions, for words used in a technical manner. For a definition to be helpful, it must be expressed in words whose meanings are already known and understood.

Compare the following definitions.

Fountain: a roundel that is barry wavy of six argent and azure.

Guige: a belt that is worn over the right shoulder and used to support a shield.

1. Which of the two definitions above is easier to understand? Why?

For a new vocabulary term to be helpful, it should be defined using words that have already been defined. The *first* definitions used in building a system, however, cannot be defined in terms of other vocabulary words because no other vocabulary words have been defined yet. In geometry, it is traditional to start with the simplest and most fundamental terms—without trying to define them—and use these terms to define other terms and develop the system of geometry. These fundamental **undefined terms** are **point**, **line**, and **plane**.

2. Define each term using the undefined terms.

a. Ray

b. Collinear points

c. Coplanar points

MATH TERMS

The term, line segment, can be defined in terms of undefined terms: A **line segment** is part of a line bounded by two points on the line called endpoints.

My Notes

ACADEMIC VOCABULARY

complementary angles
 supplementary angles
 postulate

CONNECT TO ALGEBRA

Addition Property of Equality

If $a = b$ and $c = d$,
 then $a + c = b + d$.

Subtraction Property of Equality

If $a = b$ and $c = d$,
 then $a - c = b - d$.

Multiplication Property of Equality

If $a = b$, then $ca = cb$.

Division Property of Equality

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Distributive Property

$a(b + c) = ab + ac$

Reflexive Property

$a = a$

Symmetric Property

If $a = b$, then $b = a$.

Transitive Property

If $a = b$ and $b = c$, then $a = c$.

Substitution Property

If $a = b$, then a can be substituted
 for b in any equation or inequality.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Activating Prior Knowledge

After a term has been defined, it can be used to define other terms. For example, an **angle** is defined as a figure formed by two rays with a common endpoint.

3. Define each term using the already defined terms.

a. Complementary angles

b. Supplementary angles

The process of deductive reasoning, or deduction, must have a starting point. A conclusion based on deduction cannot be made unless there is an established assertion to work from. To provide a starting point for the process of deduction, a number of assertions are accepted as true without proof. These assertions are called axioms, or **postulates**.

When you solve algebraic equations, you are using deduction. The properties you use are like postulates in geometry.

4. Using one operation or property per step, show how to solve the equation $4x + 9 = 18 - \frac{1}{2}x$. Name each operation or property used to justify each step.

The Axiomatic System of Geometry

Back to the Beginning

ACTIVITY 1.3

continued

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share

You can organize the steps and the reasons used to justify the steps in two columns with statements (steps) on the left and reasons (properties) on the right. This format is called a **two-column proof**.

EXAMPLE 1

Given: $3(x + 2) - 1 = 5x + 11$ **Prove:** $x = -3$

Statements

1. $3(x + 2) - 1 = 5x + 11$
2. $3(x + 2) = 5x + 12$
3. $3x + 6 = 5x + 12$
4. $6 = 2x + 12$
5. $-6 = 2x$
6. $-3 = x$
7. $x = -3$

Reasons

1. Given equation
2. Addition Property of Equality
3. Distributive Property
4. Subtraction Property of Equality
5. Subtraction Property of Equality
6. Division Property of Equality
7. Symmetric Property of Equality

TRY THESE A

a. Supply the reasons to justify each statement in the proof below.

Given: $\frac{x-3}{2} = \frac{6+x}{5}$

Prove: $x = 9$

Statements

1. $\frac{x-3}{2} = \frac{6+x}{5}$
2. $10\left(\frac{x-3}{2}\right) = 10\left(\frac{6+x}{5}\right)$
3. $5(x-3) = 2(6+x)$
4. $5x - 15 = 12 + 2x$
5. $3x - 15 = 12$
6. $3x = 27$
7. $x = 9$

Reasons

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

My Notes

My Notes

ACADEMIC VOCABULARY

conditional statement

WRITING MATH

The letters p and q are often used to represent the hypothesis and conclusion, respectively, in a conditional statement. The basic form of an if-then statement would then be, "If p , then q ."

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Think/Pair/Share, Interactive Word Wall, Marking the Text, Group Presentation

TRY THESE A (continued)

- b.** Complete the *Prove* statement and write a two-column proof for the equation given in Item 4. Number each statement and corresponding reason.

Given: $4x + 9 = 18 - \frac{1}{2}x$

Prove:

Rules of logical reasoning involve using a set of given statements along with a valid argument to reach a conclusion. Statements to be proved are often written in if-then form. An if-then statement is called a **conditional statement**. In such statements, the *if* clause is the **hypothesis**, and the *then* clause is the **conclusion**.

EXAMPLE 2

Conditional statement: If $3(x + 2) - 1 = 5x + 11$, then $x = -3$.	
Hypothesis	Conclusion
$3(x + 2) - 1 = 5x + 11$	$x = -3$

TRY THESE B

Use the conditional statement: If $x + 7 = 10$, then $x = 3$.

- What is the hypothesis?
- What is the conclusion?
- State the property of equality that justifies the conclusion of the statement.

The Axiomatic System of Geometry

Back to the Beginning

ACTIVITY 1.3

continued

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall

Conditional statements may not always be written in if-then form. You can restate such conditional statements in if-then form.

5. Restate each conditional statement in if-then form.

- a. I'll go if you go.
- b. There is smoke only if there is fire.
- c. $x = 4$ implies $x^2 = 16$.

An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. This type of example is a **counterexample**.

6. This is a false conditional statement.

If two numbers are odd, then their sum is odd.

- a. Identify the hypothesis of the statement.
- b. Identify the conclusion of the statement.
- c. Give a counterexample for the conditional statement and justify your choice for this example.

My Notes

READING MATH

Forms of conditional statements include:

- If p , then q .
- p only if q
- p implies q .
- q if p

ACADEMIC VOCABULARY

counterexample

My Notes

MATH TERMS

converse

inverse

contrapositive

CONNECT TO AP

When both a statement and its converse are true, you can connect the hypothesis and conclusion with the words “if and only if.”

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share

Every conditional statement has three related conditionals. These are the **converse**, the **inverse**, and the **contrapositive** of the conditional statement. The converse of a conditional is formed by interchanging the hypothesis and conclusion of the statement. The inverse is formed by negating both the hypothesis and the conclusion. Finally, the contrapositive is formed by interchanging *and* negating both the hypothesis and the conclusion.

Conditional: If p , then q .

Converse: If q , then p .

Inverse: If not p , then not q .

Contrapositive: If not q , then not p .

7. Given the conditional statement:

If a figure is a triangle, then it is a polygon.

Complete the table.

Form of the statement	Write the statement	True or False?	If the statement is false, give a counterexample.
Conditional statement	If a figure is a triangle, then it is a polygon.		
Converse of the conditional statement			
Inverse of the conditional statement			
Contrapositive of the conditional statement			

SUGGESTED LEARNING STRATEGIES: Group Presentation

If a given conditional statement is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are **logically equivalent**.

8. Write a true conditional statement whose inverse is false.

9. Write a true conditional statement that is logically equivalent to its converse.

When a statement and its converse are both true, they can be combined into one statement using the words “if and only if”. All definitions you have learned can be written as “if and only if” statements.

10. Write the definition of perpendicular lines in if and only if form.

TRY THESE C

Use this statement: *Numbers that do not end in 2 are not even.*

a. Rewrite the statement in if-then form and state whether it is true or false.

b. Write the converse and state whether it is true or false. If false, give a counterexample.

c. Write the inverse and state whether it is true or false.

d. Write the contrapositive and state whether it is true or false. If false, give a counterexample.

My Notes

MATH TERMS

The **truth value** of a statement is the truth or falsity of that statement

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Identify the property that justifies the statement: If $4x - 3 = 7$, then $4x = 10$.
2. Complete the prove statement and write a two column proof for the equation:

Given: $x - 2 = 3(x - 4)$

Prove:

3. Write the statement in if-then form.
Two angles have measures that add up to 90° only if they are complements of each other.

Use the following statement for Items 4–7.

If a vehicle has four wheels, then it is a car.

4. State the hypothesis.
5. Write the converse.
6. Write the inverse.
7. Write the contrapositive.
8. Given the false conditional statement:
If a vehicle has four wheels, then it is a car.
Write a counterexample.

9. Which of the following is a counterexample of this statement?

If an angle is acute, then it measures 80° .

- a. a 100° angle
 - b. a 70° angle
 - c. an 80° angle
 - d. a 90° angle
10. Give an example of a false statement that has a true converse.
 11. A certain conditional statement is true. Which of the following must also be true?
 - a. converse
 - b. inverse
 - c. contrapositive
 - d. all of the above
 12. Given: (1) If X is blue, then Y is gold.
(2) Y is not gold.

Which of the following *must* be true?

- a. Y is blue.
 - b. Y is not blue.
 - c. X is not blue.
 - d. X is gold.
13. **MATHEMATICAL REFLECTION** Some test questions ask you to tell if a statement is sometimes true, always true, or never true. How can counterexamples help you to answer these types of questions?